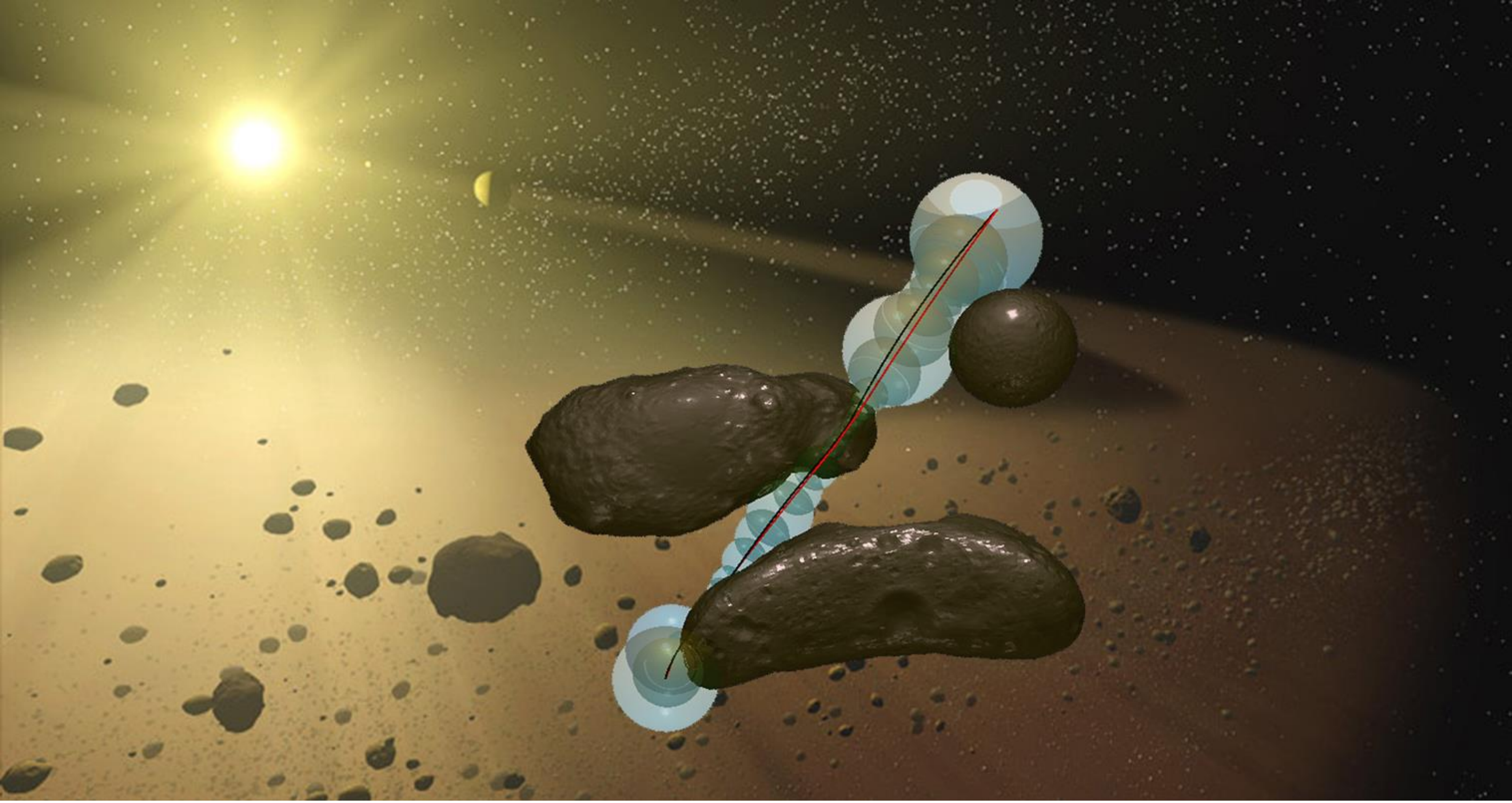


Distributed Fast Motion Planning for Spacecraft Swarms in Cluttered Environments

Saptarshi Bandyopadhyay[†], Francesca Baldini[||], Rebecca Foust[*], Soon-Jo Chung[||], Amir Rahmani[†], Jean-Pierre de la Croix[†], Fred Y. Hadaegh[†]

[†] *JPL*, [||] *Caltech*, [*] *UIUC*

Jet Propulsion Laboratory
California Institute of Technology

Motivation

- Objective: Trajectory planning of multi-agent systems (or swarms) in an obstacle rich environment
  - Navigation through debris fields, asteroid belt
  - Multi-satellite missions, docking with uncooperative targets
- Assumption: Obstacle field is known a priori
- Multi-Agent Spherical Expansion and Sequential Convex Programming (Multi-Agent SE-SCP) algorithm:
  - Trajectory is compatible with spacecraft dynamics
  - Real-time implementation
  - Guarantee any-time local optimality

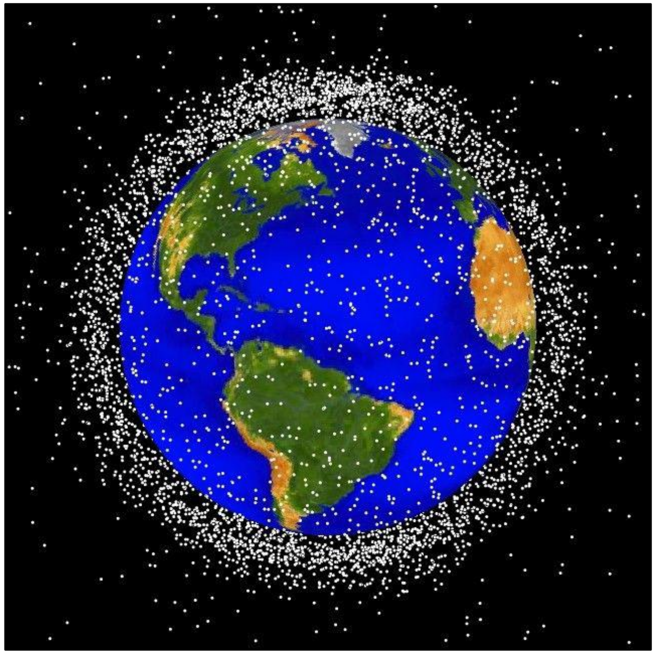
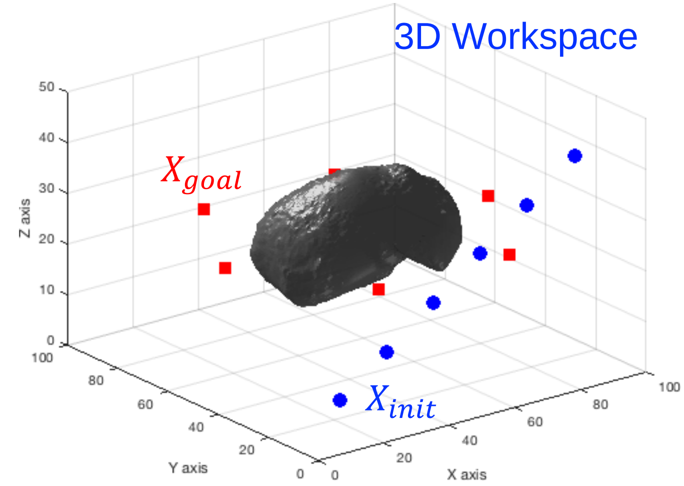


Image credit: NASA
[1] http://www.nasa.gov/mission_pages/station/news/orbital_debris.html
[2] https://www.nasa.gov/mission_pages/spitzer/multimedia/pia16610.html

Relevant Approaches in Literature



- Algorithms that discretize the workspace
  - Not suitable for incorporating dynamics
- Multi-spacecraft trajectory planning
  - Usually deals with cooperative obstacles [1]
- Sampling based algorithms in robotics
  - RRT*, PRM*, RRG work well with geometrically-fixed obstacles and are asymptotically optimal in distance-based costs [2]-[4]
  - Differential flatness technique used to incorporate dynamics [5]
  - Weak guarantees of optimality (asymptotic optimality)

[1] Morgan, et. al., "Swarm Assignment and Trajectory Optimization Using Variable-Swarm, Distributed Auction Assignment and Sequential Convex Programming," IJRR, 2016
[2] LaValle et. al., "Randomized kinodynamic planning," IJRR, 2001
[3] Kavraki, "Probabilistic roadmaps for path planning in high-dimensional configuration spaces," IEEE TRO, 1996
[4] Karaman et. al. "Sampling-based algorithms for optimal motion planning," IJRR, 2011.
[5] Kumar et. al., "Minimum snap trajectory generation and control for quadrotors," ICRA, 2011.

Prior Work: SE-SCP Algorithm

- SE-SCP solution approach has 2 steps:
  - Explore: Spherical Expansion step
  - Optimize: Sequential Convex Programming (SCP) step

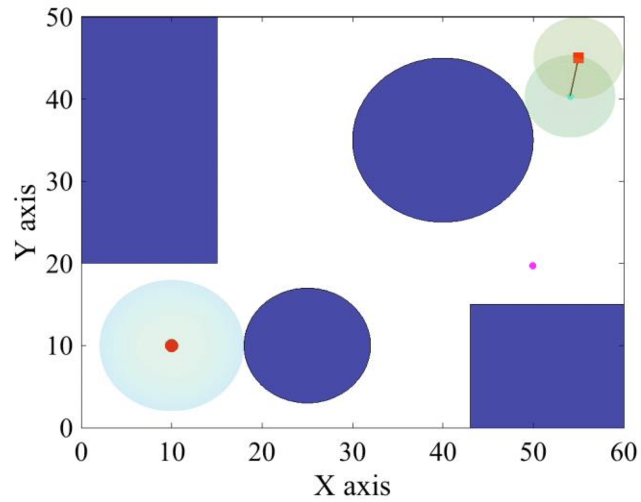
minimize cost

subject to *boundary conditions*

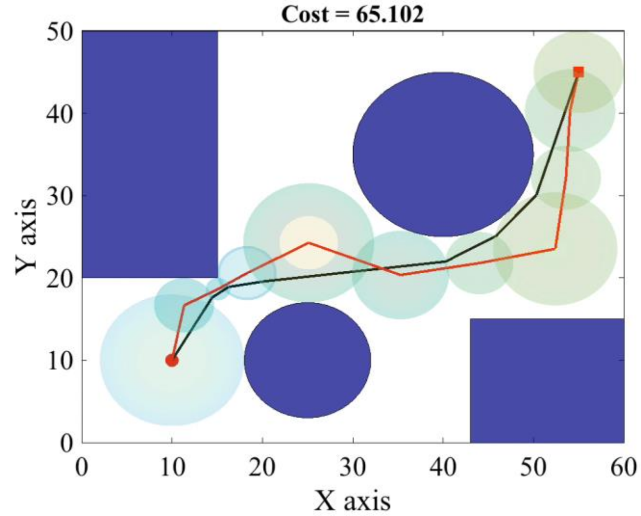
linearized dynamics

trajectory goes through spheres

- Any-time local optimality
- Asymptotic global optimality



Spherical Expansion Step



SCP Step

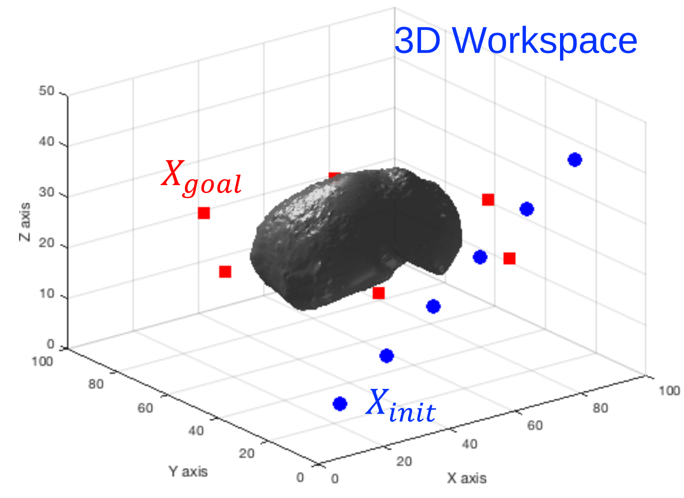
[1] F. Baldini et. al., "Fast Motion Planning for Agile Space Systems with Multiple Obstacles," AIAA/AAS Astrodynamics Specialist Conference, Long Beach, CA, September, 2016.

Problem Statement

- 3D workspace $\mathcal{X} \subset \mathbb{R}^3$ in LVLH frame
- Known stationary obstacles $\mathcal{X}_{obs} \subset \mathcal{X}$
- N agents (spacecraft)
- Initial positions $X_{init}^i \in \mathcal{X},\ \forall i \in \{1, \ldots, N\}$
- Terminal positions $X_{goal}^j \in \mathcal{X},\ \forall j \in \{1, \ldots, N\}$
- Collision avoidance constraint among agents

$$\left\| Y_k^i - Y_k^j \right\|_2 \geq r_{col}, \qquad \forall i, j \in \{1, \ldots, N\}, \qquad \forall k \in \mathbb{N}$$

- The objective of the Multi-Agent SE-SCP algorithm is to ensure that all the N agents reach the N terminal positions while avoiding collisions with the obstacles and among themselves.



Pseudo-code of Multi-Agent SE-SCP Alg.

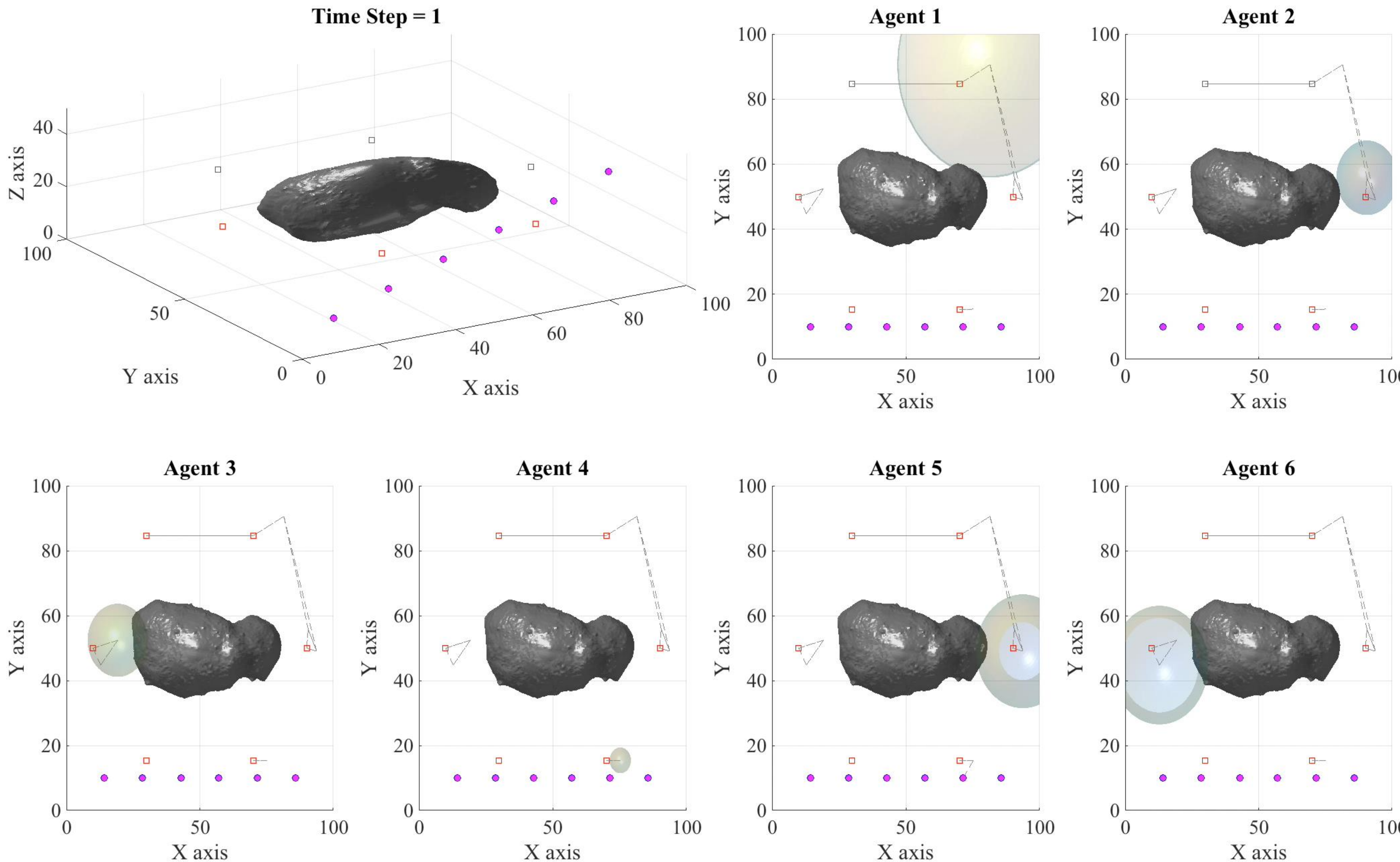
Spherical Expansion Step

Sequential Convex Programming (SCP) Step

Algorithm 1 Multi-Agent SE–SCP Algorithm for the i^{th} agent

$r^i_{\text{init}} \leftarrow \texttt{MinDistObs}(X^i_{\text{init}}, \mathcal{X}_{\text{obs}})$, $\mathcal{V}^i \leftarrow \{X^i_{\text{init}}[r^i_{\text{init}}]\}$ ▷ *Initialization step*
for $j = \{1, \ldots, N\}$ **do**
$r^j_{\text{goal}} \leftarrow \texttt{MinDistObs}(X^j_{\text{goal}}, \mathcal{X}_{\text{obs}})$, $\mathcal{V}^i \leftarrow \mathcal{V}^i \cup \{X^j_{\text{goal}}[r^j_{\text{goal}}]\}$
end for
$\mathcal{E}^i \leftarrow \emptyset$, $F^i_{\text{reached}} \leftarrow 0$, $F^i_{\text{connected}} \leftarrow 0$, $X^i_{\text{term}} \leftarrow \emptyset$
while $F_{\text{reached}} \neq 1$ **do**
$Y^\ell, X^\ell_{\text{new}}, F^\ell_{\text{connected}}, \forall \ell \in \{1, \ldots, N\} \leftarrow \texttt{AllAgentCommunicate}$ ▷ *Spherical Expansion step*
$\hat{\mathcal{X}}^i_{\text{obs}} = \mathcal{X}_{\text{obs}}$
for $\ell = \{1, \ldots, N\}/\{i\}$ **do**
$\hat{\mathcal{X}}^i_{\text{obs}} = \hat{\mathcal{X}}^i_{\text{obs}} \cup \texttt{GenerateSphere}(Y^\ell, r_{\text{col}} + r_{\text{max}})$, $\mathcal{V}^i \leftarrow \mathcal{V}^i \cup \{X^\ell_{\text{new}}[0]\}$
end for
$\mathcal{V}^i_{\text{new}} \leftarrow \emptyset$
for all $X_{\text{v}}[r_{\text{v}}] \in \mathcal{V}^i$ **do**
$r_{\text{v}} \leftarrow \texttt{MinDistObs}(X_{\text{v}}, \hat{\mathcal{X}}^i_{\text{obs}})$, $\mathcal{V}^i_{\text{new}} \leftarrow \mathcal{V}^i_{\text{new}} \cup \{X_{\text{v}}[r_{\text{v}}]\}$
end for
$\mathcal{V}^i \leftarrow \mathcal{V}^i_{\text{new}}$
$X_{\text{rand}} \leftarrow \texttt{GenerateSample}$
$X_{\text{nearest}} \leftarrow \texttt{NearestNode}(\mathcal{V}^i, X_{\text{rand}})$
$X^i_{\text{new}} \leftarrow \texttt{Steer}(X_{\text{rand}}, X_{\text{nearest}})$
$r^i_{\text{new}} \leftarrow \texttt{MinDistObs}(X^i_{\text{new}}, \hat{\mathcal{X}}^i_{\text{obs}})$, $\mathcal{V}^i \leftarrow \mathcal{V}^i \cup \{X^i_{\text{new}}[r^i_{\text{new}}]\}$
$\mathcal{E}^i \leftarrow \emptyset$
for all $X_{\text{v}}[r_{\text{v}}], X_{\text{w}}[r_{\text{w}}] \in \mathcal{V}^i$ and $X_{\text{v}} \neq X_{\text{w}}$ **do**
if $\|X_{\text{v}} - X_{\text{w}}\|_2 \leq r_{\text{v}} + r_{\text{w}}$ **then**
$c_{\text{v,w}} \leftarrow \texttt{EdgeCost}(X_{\text{v}}, X_{\text{w}})$, $c_{\text{w,v}} \leftarrow \texttt{EdgeCost}(X_{\text{w}}, X_{\text{v}})$
$\mathcal{E}^i \leftarrow \mathcal{E}^i \cup \{\overrightarrow{X_{\text{v}} X_{\text{w}}}[c_{\text{v,w}}]\} \cup \{\overrightarrow{X_{\text{w}} X_{\text{v}}}[c_{\text{w,v}}]\}$
end if
end for
if $X^i_{\text{term}} = \emptyset$ **then** ▷ *Sequential Convex Programming step*
if $\sum_{\ell=1}^N F^\ell_{\text{connected}} = N^2$ **then**
$X^i_{\text{term}} \leftarrow \texttt{DistributedAssignment}$, $\mathcal{V}^i \leftarrow \mathcal{V}^i \cup \{X^i_{\text{term}}[0]\}$
else
$F^i_{\text{connected}} \leftarrow 0$
for $j = \{1, \ldots, N\}$ **do**
$P^{i,j}, c_{P^{i,j}} \leftarrow \texttt{MinPath}(\mathcal{G}^i = (\mathcal{V}^i, \mathcal{E}^i), X^i_{\text{init}}, X^j_{\text{goal}})$
if $c_{P^{i,j}} < \infty$ **then**
$F^i_{\text{connected}} \leftarrow F^i_{\text{connected}} + 1$
end if
end for
end if
else
if $X^i_{\text{term}} = Y^i$ **then**
$F^i_{\text{reached}} \leftarrow 1$, $\boldsymbol{x}^i \leftarrow \emptyset$
else
$F^i_{\text{reached}} \leftarrow 0$
$P^i, c_{P^i} \leftarrow \texttt{MinPath}(\mathcal{G}^i = (\mathcal{V}^i, \mathcal{E}^i), Y^i, X^i_{\text{term}})$, $(\boldsymbol{x}^i_1, \boldsymbol{u}^i_1, c_{\boldsymbol{x}^i_1}) \leftarrow \texttt{OptimalTraj}(P^i)$
for $k = \{1, \ldots, N_{SCP}\}$ **do**
$P^i_k \leftarrow \texttt{GeneratePath}(\boldsymbol{x}^i_k)$, $(\boldsymbol{x}^i_{k+1}, \boldsymbol{u}^i_{k+1}, c_{\boldsymbol{x}^i_{j+1}}) \leftarrow \texttt{OptimalTraj}(P^i_k, \boldsymbol{x}^i_k, \boldsymbol{u}^i_k)$
end for
$\boldsymbol{x}^i \leftarrow \boldsymbol{x}^i_{N_{SCP}+1}$
end if
end if
$Y^i \leftarrow \texttt{AgentMotion}(\boldsymbol{x}^i)$, $\mathcal{V}^i \leftarrow \mathcal{V}^i \cup \{Y^i[0]\}$
end while

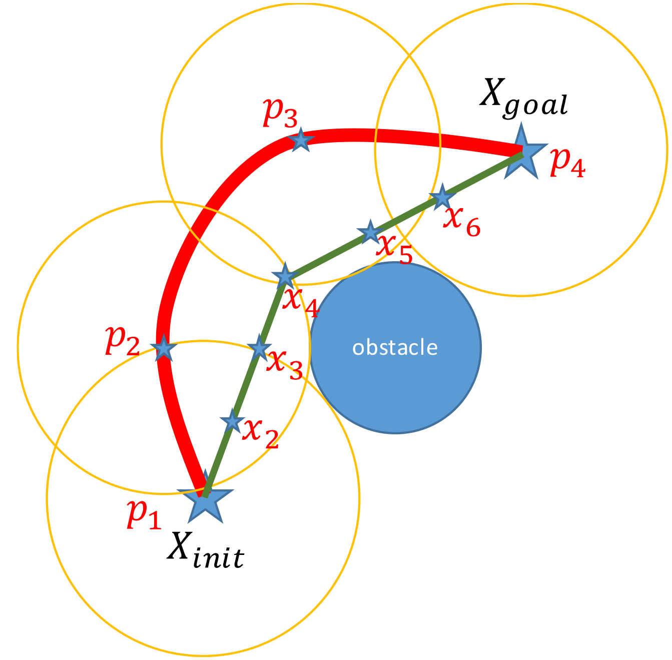
Solution of Multi-Agent SE-SCP Algorithm



Spherical Expansion Step

Sequential Convex Programming Step

Given the nodes in a path and their corresponding radii, the convex optimization problem is written as:



Problem 1: *Discrete-time Convex Optimal Motion Planning Problem*

$$\underset{\substack{\boldsymbol{x}[k],\,\forall k\in\{0,\ldots,T\}\\ \boldsymbol{u}[k],\,\forall k\in\{0,\ldots,T-1\}}}{\text{minimize}} \quad \sum_{k=0}^{T-1} c\left(\boldsymbol{u}[k]\right)\Delta\,, \tag{3}$$

minimize cost

$$\text{subject to } \boldsymbol{p}[0] = Y^i\,, \tag{4}$$

$$\boldsymbol{p}[T] = X^i_{\text{term}}\,, \tag{5}$$

boundary condition

$$\|\boldsymbol{p}[2\ell] - X_\ell\|_2 \leq r_\ell\,, \quad \forall \ell \in \{1,\ldots,n-1\}\,, \tag{6}$$

$$\|\boldsymbol{p}[2\ell] - X_{\ell+1}\|_2 \leq r_{\ell+1}\,, \quad \forall \ell \in \{1,\ldots,n-1\}\,, \tag{7}$$

$$\|\boldsymbol{p}[2\ell+1] - X_{\ell+1}\|_2 \leq r_{\ell+1}\,, \quad \forall \ell \in \{1,\ldots,n-2\}\,, \tag{8}$$

trajectory goes through spheres

$$\boldsymbol{u}[k] \in \mathcal{U}\,, \quad \forall k \in \{0,\ldots,T-1\}\,, \tag{9}$$

$$\boldsymbol{x}[k+1] = F[k]\,\boldsymbol{x}[k] + G[k]\,\boldsymbol{u}[k] + H[k]\,, \quad \forall k \in \{0,\ldots,T-1\}\,. \tag{10}$$

linearized dynamics

Sequential Convex Programming Step

Conclusions

- Multi-Agent SE-SCP algorithm:
  - Generates spacecraft trajectory through multiple geometrically-fixed obstacles
  - Two steps: the spherical expansion step and the sequential convex optimization step
  - Is any-time locally optimal and asymptotically global optimal
- Future work will focus on moving obstacles and limited FOV

Thank You